

Perturbations in Spacecraft Dynamics

Dr. Ugur GUVEN

Orbital Perturbations

- There are other forces acting on a satellite that perturb it away from the nominal orbit. These *perturbations*, or variations in the orbital elements, can be classified based on how they affect the Keplerian elements.
- *Secular variations* represent a linear variation in the element, *short-period variations* are periodic in the element with a period less than the orbital period, and *long-period variations* are those with a period greater than the orbital period. Because secular variations have long-term effects on orbit prediction (the orbital elements affected continue to increase or decrease)

Third Body Perturbations

- The gravitational forces of the Sun and the Moon cause periodic variations in all of the orbital elements, but only the longitude of the ascending node, argument of perigee, and mean anomaly experience secular variations.
- These secular variations arise from a gyroscopic precession of the orbit about the ecliptic pole. The secular variation in mean anomaly is much smaller than the mean motion and has little effect on the orbit, however the secular variations in longitude of the ascending node and argument of perigee are important, especially for high-altitude orbits.

Moon and Sun Perturbations

- For nearly circular orbits the equations for the secular rates of change resulting from the Sun and Moon are
- Longitude of the ascending node:

$$\Omega_{\text{moon}} = -0.00338 \cos(i) / n$$

$$\Omega_{\text{sun}} = -0.00154 \cos(i) / n$$

- Argument of perigee:

$$\omega_{\text{moon}} = 0.00169(4 - 5 \sin^2 i) / n$$

$$\omega_{\text{sun}} = 0.00077(4 - 5 \sin^2 i) / n$$

where i is the orbit inclination, n is the number of orbit revolutions per day, Ω and ω and are in degrees per day

Problem 13

- Calculate the perturbations in longitude of the ascending node and argument of perigee caused by the Moon and Sun for the International Space Station orbiting at an altitude of 400 km, an inclination of 51.6 degrees, and with an orbital period of 92.6 minutes.
- $i = 51.6$ degrees $n = 1436 / 92.6 = 15.5$ revolutions/day

$$\Omega_{\text{moon}} := -0.00338 \times \cos(51.6) / 15.5 = -0.000135 \text{ deg/day}$$

$$\Omega_{\text{sun}} = -0.00154 \times \cos(51.6) / 15.5 = -0.0000617 \text{ deg/day}$$

$$\omega_{\text{moon}} = 0.00169 \times (4 - 5 \times \sin^2 51.6) / 15.5 = 0.000101 \text{ deg/day}$$

$$\omega_{\text{sun}} = 0.00077 \times (4 - 5 \times \sin^2 51.6) / 15.5 = 0.000046 \text{ deg/day}$$

Perturbations due to Non-Spherical Earth

- In fact, the Earth is neither homogeneous nor spherical. The most dominant features are a bulge at the equator, a slight pear shape, and flattening at the poles. For a potential function of the Earth, we can find a satellite's acceleration by taking the gradient of the potential function. The most widely used form of the geopotential function depends on latitude and geopotential coefficients, J_n , called the *zonal coefficients*.
- The potential generated by the non-spherical Earth causes periodic variations in all the orbital elements. The dominant effects, however, are secular variations in longitude of the ascending node and argument of perigee because of the Earth's oblateness, represented by the J_2 term in the geopotential expansion.

Perturbations due to Non-Spherical Earth

$$\Omega_{J_2} = -1.5nJ_2(R_E/a)^2(\cos i)(1-e^2)^{-2}$$

$$\approx -2.06474 \times 10^{14} a^{-7/2}(\cos i)(1-e^2)^{-2}$$

$$\omega_{J_2} = 0.75nJ_2(R_E/a)^2(4-5\sin^2 i)(1-e^2)^{-2}$$

$$\approx 1.03237 \times 10^{14} a^{-7/2}(4-5\sin^2 i)(1-e^2)^{-2}$$

- where n is the mean motion in degrees/day, J_2 has the value 0.00108263, R_E is the Earth's equatorial radius, a is the semi-major axis in kilometers, i is the inclination, e is the eccentricity, and Ω and ω are in degrees/day. For satellites in GEO and below, the J_2 perturbations dominate; for satellites above GEO the Sun and Moon perturbations dominate.
- *Molniya* orbits are designed so that the perturbations in argument of perigee are zero. This condition occurs when the term $4-5\sin^2 i$ is equal to zero or, that is, when the inclination is either 63.4 or 116.6 degrees.

Problem 14

- A satellite is in an orbit with a semi-major axis of 7,500 km, an inclination of 28.5 degrees, and an eccentricity of 0.1. Calculate the J_2 perturbations in longitude of the ascending node and argument of perigee.

ANSWER

Given: $a = 7,500$ km $i = 28.5$ degrees $e = 0.1$

$$\begin{aligned} J_2 &= -2.06474 \times 10^{14} \times a^{-7/2} \times (\cos i) \times (1 - e^2)^{-2} \\ &= -2.06474 \times 10^{14} \times (7,500)^{-7/2} \times (\cos 28.5) \times (1 - (0.1)^2)^{-2} \\ &= -5.067 \text{ deg/day} \end{aligned}$$

$$\begin{aligned} J_2 &= 1.03237 \times 10^{14} \times a^{-7/2} \times (4 - 5 \times \sin^2 i) \times (1 - e^2)^{-2} \\ &= 1.03237 \times 10^{14} \times (7,500)^{-7/2} \times (4 - 5 \times \sin^2 28.5) \times (1 - (0.1)^2)^{-2} \\ &= 8.250 \text{ deg/day} \end{aligned}$$

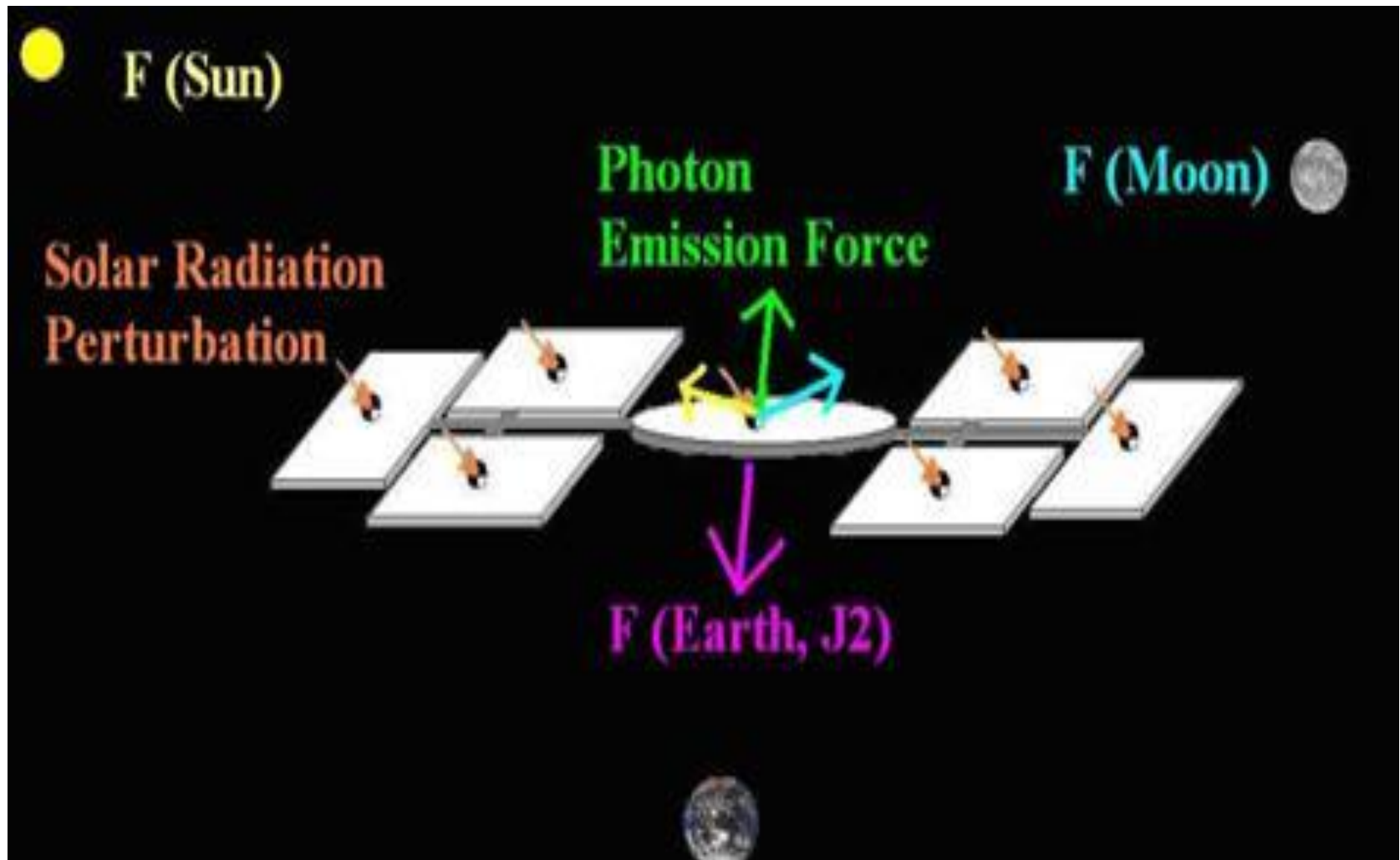
Perturbations from Solar Radiation

- Solar radiation pressure causes periodic variations in all of the orbital elements. The magnitude of the acceleration in m/s^2 arising from solar radiation pressure is:

$$a_R = \frac{-4.5 \times 10^{-8} A}{m}$$

- where A is the cross-sectional area of the satellite exposed to the Sun and m is the mass of the satellite in kilograms.
- For satellites below 800 km altitude, acceleration from atmospheric drag is greater than that from solar radiation pressure; above 800 km, acceleration from solar radiation pressure is greater.

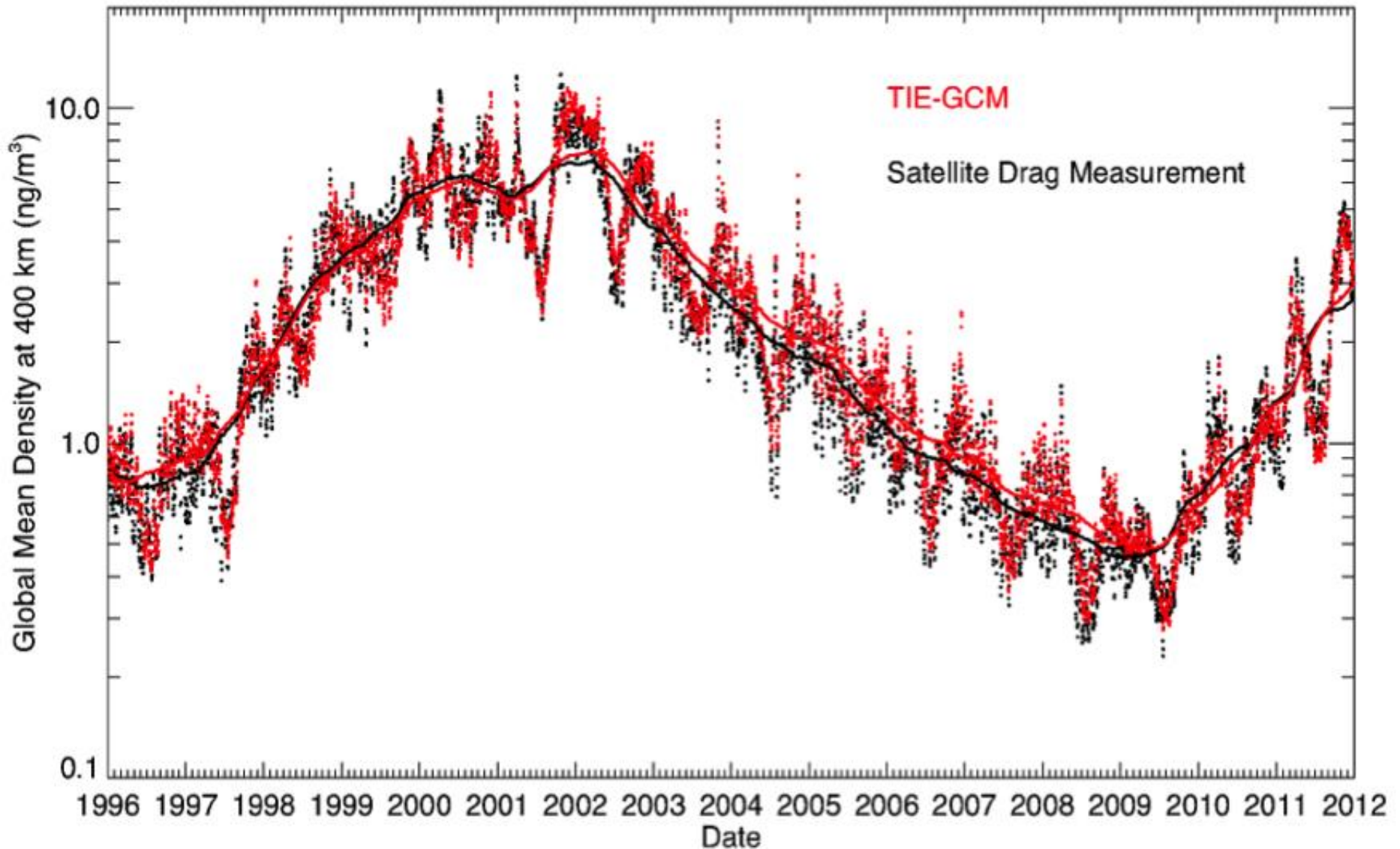
Solar Perturbation



Perturbations from Atmospheric Drag

- Drag is the resistance offered by a gas or liquid to a body moving through it. A spacecraft is subjected to drag forces when moving through a planet's atmosphere. This drag is greatest during launch and reentry, however, even a space vehicle in low Earth orbit experiences some drag as it moves through the Earth's thin upper atmosphere.
- In time, the action of drag on a space vehicle will cause it to spiral back into the atmosphere, eventually to disintegrate or burn up. If a space vehicle comes within 120 to 160 km of the Earth's surface, atmospheric drag will bring it down in a few days, with final disintegration occurring at an altitude of about 80 km. Above approximately 600 km, on the other hand, drag is so weak that orbits usually last more than 10 years - beyond a satellite's operational lifetime. The deterioration of a spacecraft's orbit due to drag is called *decay*.

Atmospheric Drag on a Satellite



Drag Force on a Body

- The drag force F_D on a body acts in the opposite direction of the velocity vector and is given by the equation:

$$F_D = \frac{1}{2} C_D \rho v^2 A$$

- where C_D is the drag coefficient, ρ is the air density, v is the body's velocity, and A is the area of the body normal to the flow. The drag coefficient is dependent on the geometric form of the body and is generally determined by experiment.
- Earth orbiting satellites typically have very high drag coefficients in the range of about 2 to 4.

Satellite Decay Due to Solar Activity

- The region above 90 km is the Earth's *thermosphere* where the absorption of extreme ultraviolet radiation from the Sun results in a very rapid increase in temperature with altitude. At approximately 200-250 km this temperature approaches a limiting value, the average value of which ranges between about 600 and 1,200 K over a typical solar cycle.
- Solar activity also has a significant effect on atmospheric density, with high solar activity resulting in high density. Below about 150 km the density is not strongly affected by solar activity; however, at satellite altitudes in the range of 500 to 800 km, the density variations between solar maximum and solar minimum are approximately two orders of magnitude.
- The large variations imply that satellites will decay more rapidly during periods of solar maxima and much more slowly during solar minima.

Decay Analysis for Circular Orbits

- For circular orbits we can approximate the changes in semi-major axis, period, and velocity per revolution using the following equations:

$$\Delta a_{\text{rev}} = \frac{-2\pi C_D A P a^2}{m}$$

$$\Delta P_{\text{rev}} = \frac{-6\pi^2 C_D A P a^2}{mV}$$

$$\Delta V_{\text{rev}} = \frac{\pi C_D A P a V}{m}$$

- where a is the semi-major axis, P is the orbit period, and V , A and m are the satellite's velocity, area, and mass respectively. The term $m/(C_D A)$, called the *ballistic coefficient*, is given as a constant for most satellites. Drag effects are strongest for satellites with low ballistic coefficients, this is, light vehicles with large frontal areas.

Satellite Lifetime Due to Drag

- A rough estimate of a satellite's lifetime, L , due to drag can be computed from:

$$L \approx \frac{-H}{\Delta a_{\text{rev}}}$$

- where H is the atmospheric density scale height.

Atmospheric Properties

Physical Properties of U.S. Standard Atmosphere, 1976 in SI Units

Altitude (meters)	Temperature (K)	Pressure (Pa)	Density (kg/m ³)	Viscosity (N·s/m ²)
-2,000	301.15	1.27774E+5	1.47808	1.87630E-5
-1,000	294.65	1.13929E+5	1.34700	1.84434E-5
0	288.15	1.01325E+5	1.22500	1.81206E-5
1,000	281.65	8.98746E+4	1.11164	1.77943E-5
2,000	275.15	7.94952E+4	1.00649	1.74645E-5
3,000	268.65	7.01085E+4	9.09122E-1	1.71311E-5
4,000	262.15	6.16402E+4	8.19129E-1	1.67940E-5
5,000	255.65	5.40199E+4	7.36116E-1	1.64531E-5
6,000	249.15	4.71810E+4	6.59697E-1	1.61084E-5
7,000	242.65	4.10607E+4	5.89501E-1	1.57596E-5
8,000	236.15	3.55998E+4	5.25168E-1	1.54068E-5
9,000	229.65	3.07425E+4	4.66348E-1	1.50498E-5
10,000	223.15	2.64363E+4	4.12707E-1	1.46884E-5
12,000	216.65	1.93304E+4	3.10828E-1	1.43226E-5
15,000	216.65	1.20446E+4	1.93674E-1	1.43226E-5
20,000	216.65	5.47489E+3	8.80349E-2	1.43226E-5
25,000	221.65	2.51102E+3	3.94658E-2	1.46044E-5
30,000	226.65	1.17187E+3	1.80119E-2	1.48835E-5
35,000	237.05	5.58924E+2	8.21392E-3	1.54559E-5
40,000	251.05	2.77522E+2	3.85101E-3	1.62096E-5
45,000	265.05	1.43135E+2	1.88129E-3	1.69449E-5
50,000	270.65	7.59448E+1	9.77525E-4	1.72341E-5
60,000	245.45	2.03143E+1	2.88321E-4	1.59104E-5
70,000	217.45	4.63422	7.42430E-5	1.43679E-5
80,000	196.65	8.86280E-1	1.57005E-5	1.31682E-5
84,852	186.95	3.73384E-1	6.95788E-6	1.25915E-5

Basic Assumptions

Air is a clean, dry, perfect gas mixture; Specific heat ratio = 1.40; Molecular weight to 86 km = 28.9644

Principal sea-level constituents: N₂-78.084%, O₂-20.9476%, Ar-0.934%, CO₂-0.0314%, Ne-0.001818%, He-0.000524%, CH₄-0.0002%

Problem 15

A satellite is in a circular Earth orbit at an altitude of 400 km. The satellite has a cylindrical shape 2 m in diameter by 4 m long and has a mass of 1,000 kg. The satellite is traveling with its long axis perpendicular to the velocity vector and its drag coefficient is 2.67. Calculate the perturbations due to atmospheric drag and estimate the satellite's lifetime.

SOLUTION,

Given:

$$a = (6,378.14 + 400) \times 1,000 = \underline{6,778,140 \text{ m}}$$

$$A = 2 \times 4 = 8 \text{ m}^2$$

$$m = 1,000 \text{ kg}$$

$$C_D = 2.67$$

- From [Atmosphere Properties](#),

$$\rho = 2.62 \times 10^{-12} \text{ kg/m}^3$$

$$H = 58.2 \text{ km}$$

- $V = \text{SQRT}[GM / a]$

$$V = \text{SQRT}[3.986005 \times 10^{14} / 6,778,140]$$

$$V = \underline{7,669 \text{ m/s}}$$

Use Equations given for decay analysis in circular orbit

$$\Delta a_{\text{rev}} = (-2 \times \pi \times C_D \times A \times \rho \times a^2) / m$$

$$\Delta a_{\text{rev}} = (-2 \times \pi \times 2.67 \times 8 \times 2.62 \times 10^{-12} \times 6,778,140^2) / 1,000$$

$$\Delta a_{\text{rev}} = \underline{-16.2 \text{ m}}$$

$$\Delta P_{\text{rev}} = (-6 \times \pi^2 \times C_D \times A \times \rho \times a^2) / (m \times V)$$

$$\Delta P_{\text{rev}} = (-6 \times \pi^2 \times 2.67 \times 8 \times 2.62 \times 10^{-12} \times 6,778,140^2) / (1,000 \times 7,669)$$

$$\Delta P_{\text{rev}} = \underline{-0.0199 \text{ s}}$$

$$\Delta V_{\text{rev}} = (\pi \times C_D \times A \times \rho \times a \times V) / m$$

$$\Delta V_{\text{rev}} = (\pi \times 2.67 \times 8 \times 2.62 \times 10^{-12} \times 6,778,140 \times 7,669) / 1,000$$

$$\Delta V_{\text{rev}} = \underline{0.00914 \text{ m/s}}$$

Equation (for drag is implemented

$$L \sim -H / \Delta a_{\text{rev}}$$

$$L \sim -(58.2 \times 1,000) / -16.2 \quad (\times 1000 \text{ because of meter conversion})$$

$$L \sim \underline{3,600 \text{ revolutions}}$$

Perturbation Summary

Disturbing Forces

